$$(1).\ p=\frac{1}{(\frac{4}{3}\pi r^3)\pi r^2}\int_0^r\int_0^r\int_{-y}^{z'}2\pi x dx.2\pi y dy dz=\frac{3}{2r^5}\int_0^r\int_0^r y[r^2-(x-y)^2]dxdy$$

$$=\frac{1}{8r^5}\int_0^r(3r^4-6r^2x^2+8r^3x)dx=\tfrac{5}{8}.$$

$$(2).\ \Delta=\frac{1}{(\frac{4}{3}\pi r^3)\pi r^2}\int_0^r 2\pi x dx\Big[\int_0^x\int_{-y}^{y}2\pi y\surd(x^2+y^2+2xz)dydz$$

$$+\int_x^r\int_{-y}^{y}2\pi y\surd(x^2+y^2+2xz)dydz\Big].$$

$$\therefore\ \Delta=\frac{1}{r^5}\int_0^r dx\Big[\int_0^x y[(x+y)^3-(x-y)^3]dy+\int_x^r y[(y+x)^3-(y-x)^3]dy\Big]$$

$$=\frac{1}{10r^5}\int_0^r(15r^4x+10r^2x^3-x^4)dx=\tfrac{59}{60}r.$$

Also solved by the *PROPOSER*.

105. Proposed by L. C. WALKER, Assistant in Mathematics, Leland Stanford Jr. University, Palo Alto, Cal.

Find the average distance of the center of an ellipsoid, axes $2a$, $2b$, and $2c$ from its surface.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Different interpretations will result in different solutions. The method generally employed by the best thinkers in this department of mathematics is the following:

$$d=\frac{\int r ds}{\int ds},$$ where ds is an element of surface.

Let s be the eighth part of the surface of the ellipsoid.

Then $d=\frac{1}{s}\int r ds$.

$$ds=\sqrt{1+\left(\frac{dz}{dx}\right)^2+\left(\frac{dz}{dy}\right)^2}dxdy=\left[\frac{a^4b^4-b^4(a^2-c^2)x^2-a^4(b^2-c^2)y^2}{a^2b^2(a^2b^2-b^2x^2-a^2y^2)}\right]^{\frac{1}{2}}dxdy$$

$$r=\surd(x^2+y^2+z^2)=\frac{1}{ab}[a^2b^2c^2+b^2(a^2-c^2)x^2+a^2(b^2-c^2)y^2]^{\frac{1}{2}}.$$

$$\therefore\ p=\int_0^a\int_0^{(b/a)\surd(a^2-x^2)}[c^2+x^2+y^2-\frac{c^2}{b^4}(b^2-c^2)y^2-\frac{c^2}{a^4}(a^2-c^2)x^2$$

$$+\frac{c^2(x^2+y^2)(b^4x^2+a^4y^2)}{a^2b^2(a^2b^2-b^2x^2-a^2y^2)}]^{\frac{1}{2}}\,dxdy.$$

This expression does not seem to be easy to integrate.

MISCELLANEOUS.

96. **Proposed by H. M. CASH, Lore City, Guernsey County, Ohio.**

A stick of timber is 12 feet long, 8 inches deep, and 3 inches wide at one end, and 5 inches deep, and 12 inches wide at the other end. An what distance from either end should it be cut to divide it into two equal parts?

Solution by H. C. WHITAKER, A.M., Ph.D., Manual Training School, Philadelphia, Pa., and G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.

By the Prismoidal Formula, Volume$=\frac{1}{6}$(upper base+lower base+4 middle section)$\times$height.

Hence, the volume of the whole stick equals

$$\tfrac{1}{6}(8\times3+5\times12+4\times\tfrac{13}{2}\times\tfrac{15}{2})\times144=6696.$$

Denote the distance of the required section from the 8×3 end by x, the depth at that section by y, the width by z. Then $y=8-\frac{1}{48}x$, and $z=3+\frac{1}{16}x$; also

$$\tfrac{1}{6}\left[8\times3+yz+4\left(\frac{8+y}{2}\right)\left(\frac{3+z}{2}\right)\right]x=3348.$$

Whence $x^3-504x^2-55296x+7713792=0$.

$x=84.8856$ inches$=7$ feet, 0.8856 inches.

Also solved by *R. L. MOORE.*

97. **Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.**

A spherical soap-bubble is electrified in such a way that the excess of the internal over the external air pressure is 2π when the bubble is in equilibrium. How does the tension of the film vary with the electric density?

Solution by the PROPOSER.

Let p be the excess of internal over external pressure, σ=electrical density, t=tension, r=radius of bubble.

Then $p+2\pi\sigma^2=2t/r$, (see Minchin, Vol. II, page 487, third edition).

But $p=2\pi$.

$\therefore 2\pi r(1+\sigma^2)=2t$, or $\dfrac{t}{1+\sigma^2}=\pi r$.